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Risk Premiums in Higher-Order Moments and Stock Return Predictability

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Abstract

In this paper, we study risk premiums in higher order moments of financial asset returns in a general equilibrium setting. Extending the model proposed by Drechsler and Yaron[2011] with a stochastic jump intensity in the processes of both the long-run risk factor and the variance of consumption growth rate, we provide explicit representations for the variance and skewness risk premiums in the equilibrium. Moreover, modeling the stochastic jump intensity endogeneously and deriving a representation of the risk-neutral skewness with that intensity, we propose a possible reason of the empirical fact of time-varying and negative risk-neutral skewness. Finally, providing an equity risk premium representation of a linear factor pricing model with the variance and skewness risk premiums, we show an empirical evidence in which the skewness risk premium, as well as the variance risk premium, has superior predictive power for future aggregate stock market index returns.

Keywords: long-run risk model, Epstein-Zin preferences, variance risk premium, skewness risk premium, stock return predictability, stochastic volatility, volatility of volatility, jump intensity
JEL Classification: C, D, G

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1 Introduction

The concern with the information content in option-implied distributions has been growing for the last several years. In particular, there has been a renewal of interest in the information of the difference between option-implied and realized distributions, which is usually recognized as a risk premium required by the representative agent, in terms of the financial risk management or the asset pricing implications. This paper examines such risk premiums in higher order moments of financial asset returns in a general equilibrium setting. In particular, we provide an equity risk premium representation of a linear factor pricing model with the risk premiums in higher order moments such as the variance and skewness risk premiums in an equilibrium, and based on that representation the stock return predictability is examined with the data of S & P500 index returns. We document the large and statistically significant predictive power of the skewness premium as well as the variance risk premium for the stock index returns.

In recent years, there remains an ever-increasing interest and challenge to develop an entirely self-contained equilibrium-based explanation for the nonzero variance risk premium and its predictability for stock index returns. To the best of our knowledge, the first attempt to demonstrate the existence of the variance risk premium in an equilibrium market is made by Bansal and Yaron[2004]. They develop the long-run risks (LRR) model which emphasizes the role of long-run risks, that is, low-frequency movements in consumption growth rates and volatility, in accounting for a wide range of asset pricing puzzles. The LRR model features an Epstein and Zin[1989] utility function with an investor preference for early resolution of uncertainty and contains (i) a persistent expected consumption growth component and (ii) long-run variation in consumption volatility. Based on the LRR model, Bansal and Yaron[2004] provide the equation for the equity premium in which have two sources of systematic risk: the first relates to fluctuations in expected consumption growth and the second to fluctuations in consumption volatility. They show that the market compensation for stochastic volatility risk in consumption, that is, the volatility risk premium, exists and appears explicitly in that equation for the equity premium.

Motivated by the implications from Bansal and Yaron[2004], a stylized self-contained general equilibrium model incorporating the effects of time-varying economic uncertainty, Bollerslev, Tauchen, and Zhou[2009] show that the difference between implied and realized variation, or the variance risk premium, is able to explain a nontrivial fraction of the time-series variation in post-1990 aggregate stock market returns, with high (low) premia predicting high (low) future returns. The magnitude of the predictability is particularly strong at the intermediate quarterly return horizon, where it dominates that afforded by other popular predictor variables, such as the P/E ratio, the default spread, and the consumption-wealth ratio.

Drechsler and Yaron[2011] also show the predictability of the variance risk premium for stock index returns based on an equilibrium model with jumps in uncertainty and the long-run component of cash-flows. They demonstrate that a risk aversion greater than one and a preference for early resolution of uncertainty correctly signs the variance risk premium and the coefficient from a predictive regression of returns on the variance risk premium.

Whereas these studies cited above are essentially based on discrete-time models, several studies such as Eraker[2008], Branger and Volkert[2010], and Bollerslev, et. al.[2012] also provide an entirely self-contained equilibrium-based explanation for the nonzero variance risk premium based on continuous-time models.

All the above studies focus only on the variance risk premium required by a representative investor due to the stochastic nature of asset return variance, and as far as we know, there are few reports about the risk premium which compensates for uncertainty of the third moment, that is, the skewness, of asset returns. In this paper, we demonstrate that the skewness risk premium, defined as the difference between option-implied and realized skewness, also captures attitudes toward economic uncertainty as well as the variance risk premium. Among recent studies on self-contained equilibrium-based models for the nonzero variance risk premium referred above, all the studies except for Drechsler and Yaron[2011] model the processes of both the variance of consumption growth rate and the LRR factor as conditional normal, so that the one-step-ahead conditional distribution of the market return is also normal and, as a result, the skewness of that distribution is zero. Therefore, those of models proposed by these studies can not explain the *negative* risk-neutral skewness which is found by the previous studies such as Aït-Sahalia and Lo[1998] and Aït-Sahalia, Wang, and Yared[2001]. They document several empirical features of the state price density for the S & P500 index option market over time, including the term structures of mean returns, volatility, skewness, and kurtosis, that are implied by option-implied distributions. In particular, They show that the nonparametric state price densities are negatively skewed, have fatter tails and the amount of skewness and kurtosis both increase with maturity.

We show that jump components in the LRR factor and/or the variance of consumption growth rate can explain the nonzero (or negative) skewness of the one-step-ahead asset return distribution. To the best of our knowledge, Drechsler and Yaron[2011] is the first paper that indicates an important role for transient non-Gaussian shocks (jumps) to fundamentals such as the LRR-factor and the variance of consumption growth rate for understanding how perceptions of economic uncertainty and cash-flow risk manifest themselves in asset prices. However, the assumption of an affine structure on the jump intensity process λ_t, that is, $\lambda_t = l_0 + l_1\sigma_t^2$ where $l_0, l_1 > 0$ and σ_t^2 is the variance of consumption growth rate, in Drechsler and Yaron[2011] can not explain an empirical fact on a simultaneous relation between monthly stock returns and monthly changes of the option-implied skewness:

$$
\begin{aligned}
r_{m,t+1} &= 0.006 - 0.019 \times \triangle ISkew_{t+1} \\
&\qquad (2.46) \quad (-3.46) \\
r_{m,t+1} &= 0.006 - 0.007 \times \triangle VIX_{t+1} - 0.016 \times \triangle ISkew_{t+1} \\
&\qquad (3.33) \quad (-16.00) \qquad\qquad (-3.94)
\end{aligned}
\tag{1}
$$

where $r_{m,t+1}$ is the monthly return of the S & P500 Total Return Index from time t to $t+1$, $\triangle VIX_{t+1}$ is the monthly change of implied volatility calculated with the CBOE's VIX from time t to $t+1$, and $\triangle ISkew_{t+1}$ is the monthly change of implied skew calculated with the CBOE's Skew Index from time t to $t+1$ and these results are obtained based on the monthly data from Jan-1990 to Aug-2012. Under such assumption on the jump intensity process in Drechsler and Yaron[2011], however, we can confirm that the regression parameters to $\triangle ISkew_{t+1}$ in the above regression models should be positive.

In this paper, we propose an extension of the LRR models of Bansal and Yaron[2004] and Drechsler and Yaron[2011]. Our model contains a rich set of transient dynamics and can quantitatively account for the time variation and asset return predictability of the skewness premium as well as the variance risk premium. In particular, we introduce a stochastic jump intensity structure for transient jumps to fundamentals such as the LRR factor and the variance of consumption growth rate and show that this additional introduction of a stochastic jump intensity model enables our model proposed in this paper to capture the various empirical aspects of the stock index returns and its option implied moments including the result of (1). Christoffersen, et.al.[2012] find very strong support for time-varying jump intensities for S & P500 index returns, and they show that, compared to the risk premium on dynamic volatility, the risk premium on the dynamic jump intensity has a much larger impact on option prices. We find that the existence of the negative skewness and the skewness risk premium have a close relationship with the existence of the jumps and the jump risk premium, respectively.

This paper also shows that the skewness of asset return distribution and the skewness risk premium which compensates for the stochastic nature of the skewness are both time-varying due to the stochastic nature of the jump intensity for transient jumps in the LRR factor and the variance of consumption growth rate. Providing an equity risk premium representation of a linear factor pricing model with time-varying variance and skewness risk premiums, we find that these risk premiums can explain a nontrivial fraction of the time series variation in the aggregate stock market returns and show an empirical evidence in which the skewness risk premium, as well as the variance risk premium, has superior predictive power for future aggregate stock market index returns.

The remainder of this paper is organized as follows. Section 2 outlines the basic theoretical model with jumps in consumption growth rate and its volatility, shows how the equilibrium is derived for our model economy, and highlights its key features. In particular, we provide an equity risk premium representation of a linear factor pricing model with time-varying variance and skewness risk premiums. Section 3 provides the implications from a calibrated version of the theoretical equity risk premium representation of a linear factor pricing model derived in Section 2 to help guide and interpret our subsequent empirical reduced form predictability regressions. Section 4 describes the data used for examining that equity risk premium representation empirically and discusses the results from the predictive regressions on the stock returns to the variance and the skewness risk premiums with historical data. Section 5 provides concluding remarks.

2 Model Framework

2.1 Model Setup and Assumptions

The underlying environment is a discrete time endowment economy. The representative agent's preferences on the consumption stream are of the Epstein and Zin[1989] form, allowing for the separation of risk aversion and the intertemporal elasticity of substitution (IES). Thus, the agent maximizes his lifetime utility, which is defined recursively as

$$V_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\Big(\mathbb{E}_t[V_{t+1}^{1-\gamma}]\Big)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{2}$$

where C_t is consumption at time t, $0 < \delta < 1$ reflects the agent's time preference, γ is the coefficient of risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and ψ is the intertemporal elasticity of substitution (IES). This preference structure collapses to a standard CRRA utility representation if $\gamma = \frac{1}{\psi}$, that is, $\theta = 1$, and in this case, only innovations to consumption are priced. In the following, based on the result provided by Bansal and Yaron[2004] we assume that both γ and ψ are larger than one. It then holds that $\gamma > \frac{1}{\psi}$, which implies $\theta < 0$. With this choice, the investor has a preference for early resolution of uncertainty. Then, not only consumption risk is priced, but state variables carry risk premia, too. The parameter restrictions also ensure that the signs of the risk premia are in line with economic intuition, and that a worsening of economic conditions leads to a decrease in asset prices.

Utility maximization is subject to the budget constraint

$$W_{t+1} = (W_t - C_t)R_{c,t+1},$$

where W_t is the wealth of the agent and $R_{c,t}$ is the return on all invested wealth. As shown in Epstein and Zin[1989], for any asset j, the first-order condition yields the following Euler condition:

$$\mathbb{E}_t\Big[\exp(m_{t+1} + r_{j,t+1})\Big] = 1, \tag{3}$$

where $r_{j,t+1}$ is the log of the gross return on asset j and m_{t+1} is the log of the intertemporal marginal rate of substitution (IMRS), which is given by $m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \triangle c_{t+1} + (\theta - 1) r_{c,t+1}$. Here, $r_{c,t+1}$ is $\log R_{c,t+1}$ and $\triangle c_{t+1}$ is the change in $\log C_t$, that is, $\log \left(\frac{C_{t+1}}{C_t} \right)$.

We model consumption and dividend growth rates, $g_{t+1} \equiv \log(\frac{C_{t+1}}{C_t})$ and $g_{d,t+1} \equiv \log(\frac{D_{t+1}}{D_t})$ where D_t is dividend at time t, respectively, as containing a small persistent predictable component x_t, which determines the conditional expectation of consumption growth,

$$\begin{aligned} x_{t+1} &= \rho_x x_t + \varphi_e \sigma_t e_{t+1} + J_{x,t+1}, \\ g_{t+1} &= \mu_g + x_t + \varphi_\eta \sigma_t \eta_{t+1}, \\ g_{d,t+1} &= \mu_d + \rho_d x_t + \varphi_\zeta \sigma_t \zeta_{t+1}, \end{aligned} \tag{4}$$

where $\varphi_e, \varphi_\eta, \varphi_\xi, \rho_x, \rho_d > 0$, $\mu_g, \mu_d \in \mathbb{R}$, e_t, η_t, and ξ_t are mutually independent $i.i.d.N(0,1)$ processes, and $J_{x,t+1}$ is a compound-Poisson process represented by $J_{x,t+1} \equiv \sum_{j=1}^{N_{t+1}^x} \epsilon_x^j$ where N_{t+1}^x is the Poisson counting process for that jump component whose the intensity process is given by $\lambda_{x,t+1} \equiv l_x \lambda_{t+1}, l_x > 0$, and $\epsilon_x^j \sim i.i.d.\ N(0, \delta_x^2)$, $\delta_x > 0$, is the size of the jump that occurs upon the N_{t+1}^x.

Furthermore, we also model the dynamics of the volatility as follows:

$$\begin{aligned} \sigma_{t+1}^2 &= \mu_\sigma + \rho_\sigma \sigma_t^2 + \sqrt{q_t} w_{t+1} + J_{\sigma^2,t+1}, \\ q_{t+1} &= \mu_q + \rho_q q_t + \varphi_\xi \sqrt{q_t} \xi_{t+1}, \end{aligned} \tag{5}$$

where the parameters satisfy $\mu_\sigma > 0$, $\mu_q > 0$, $|\rho_\sigma| < 1$, $|\rho_q| < 1$, $\varphi_\xi > 0$, and w_t and ξ_t are mutually independent $i.i.d.N(0,1)$ processes and are independent of each of e_t, η_t, and ξ_t. $J_{\sigma^2,t+1}$ is a compound-Poisson process, which is represented by $J_{\sigma^2,t+1} \equiv \sum_{j=1}^{N_{t+1}^{\sigma^2}} \epsilon_{\sigma^2}^j$ where $N_{t+1}^{\sigma^2}$ is the Poisson counting process for that jump component whose the intensity process is given by $\lambda_{\sigma^2,t+1} \equiv l_{\sigma^2} \lambda_{t+1}, l_{\sigma^2} > 0$, and $\epsilon_{\sigma^2}^j \sim i.i.d.\ N(0, \delta_{\sigma^2}^2)$, $\delta_{\sigma^2} > 0$, is the size of the jump that occurs upon the $N_{t+1}^{\sigma^2}$. We assume that N_{t+1}^x and $N_{t+1}^{\sigma^2}$ are mutually independent and ϵ_x^j and $\epsilon_{\sigma^2}^j$ are too. The stochastic volatility process σ_t^2 represents time-varying economic uncertainty in consumption growth with the variance-of-variance process q_t in effect inducing an additional source of temporal variation in that same process. We also model the variance-of-variance process q_t in the same fashion as Bollerslev, et. al.[2009].

Importantly, we assume that the jump intensity dynamics in the economy is governed by the following discrete-time stochastic process,

$$\lambda_{t+1} = \mu_\lambda + \rho_\lambda \lambda_t + \varphi_u \sqrt{q_t} (\rho \xi_{t+1} + \sqrt{1-\rho^2} u_{t+1}), \tag{6}$$

where $\mu_\lambda > 0$, $|\rho_\lambda| < 1$, $|\rho| \leq 1$, and u_t is an $i.i.d.N(0,1)$ process, which is independent of each of e_t, η_t, ξ_t, w_t, and ξ_t.

One of the notable features of our model setup is this introduction for the jump intensity process (6). Christoffersen, et.al.[2012] also find very strong support for time-varying jump intensities for S & P500 index returns, and they show that, compared to the risk premium on dynamic volatility, the risk premium on the dynamic jump intensity has a much larger impact on option prices. In the area of studying on the risk premiums in higher-order moments of financial asset returns in equilibrium, although Drechsler and Yaron[2011] is the first paper that introduces transient jumps to fundamentals such as the LRR-factor x_t and the variance of consumption growth rate σ_t^2, however, it assumes that the jump intensity process λ_t is represented by an affine structure such as $\lambda_t = l_0 + l_1\sigma_t^2$ where $l_0, l_1 > 0$. As mentioned in the introduction of this paper, such assumption for the jump intensity process can not explain the empirical fact of regression (1). We extend the LRR models of Bansal and Yaron[2004] and Drechsler and Yaron[2011] so as to introduce a stochastic jump intensity of (6) into the economy. As shown in the following, this introduction enables our model to have a consistency with the empirical fact shown in (1).

2.2 The Model Solution in Equilibrium

We distinguish between the unobservable return on a claim to aggregate consumption, $R_{c,t+1}$, and the observable return on the market portfolio, $R_{m,t+1}$: the latter is the return on the aggregate dividend claim. Solving our model numerically, we demonstrate the mechanisms working in our model via approximate analytical solutions in the same fashion as the previous studies such as Bansal and Yaron[2004], Bollerslev, et.al.[2009], Drechsler and Yaron[2011], etc. To derive these solutions for our model, we use the standard approximation utilized in Campbell and Shiller (1988),

$$r_{c,t+1} = \kappa_0 + \kappa_1 v_{t+1} - v_t + g_{t+1}, \tag{7}$$

where lowercase letters refer to logs, so that $r_{c,t+1} = \log(R_{c,t+1})$ is the continuous return, $v_t = \log(\frac{P_t}{C_t})$ is the log price-consumption ratio of the asset that pays the consumption endowment, $\{C_{t+i}\}_{i=1}^{\infty}$, and κ_0 and κ_1 are approximating constants that both depend only on the average level of v [1]. Analogously, $r_{m,t+l}$ and $v_{m,t+1}$ correspond to the market return and its log price-dividend ratio and the similar approximation presented below can also be derived:

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} v_{m,t+1} - v_{m,t} + g_{d,t+1}. \tag{8}$$

The standard solution method for finding the equilibrium in a model like the one defined above then consists in conjecturing solutions for v_t and $v_{m,t}$ as an affine function

[1]Note that $\kappa_1 = \frac{\exp(\bar{v})}{1+\exp(\bar{v})}$ and κ_1 is approximately 0.997 (cf) Bansal and Yaron[2004]), which is also consistent with magnitudes used in Campbell and Shiller[1988].

of the state variables, x_t, σ_t^2, q_t, and λ_t,

$$v_t = A_0 + A_x x_t + A_\sigma \sigma_t^2 + A_q q_t + A_\lambda \lambda_t, \tag{9}$$

$$v_{m,t} = A_{0,m} + A_{x,m} x_t + A_{\sigma,m} \sigma_t^2 + A_{q,m} q_t + A_{\lambda,m} \lambda_t, \tag{10}$$

respectively, solving for the coefficients A_0, A_x, A_σ, A_q, and A_λ in v_t and for the coefficients $A_{0,m}$, $A_{x,m}$, $A_{\sigma,m}$, $A_{q,m}$, and $A_{\lambda,m}$ in $v_{m,t}$.

Substituting (9) for (7), we have a temporal representation for $r_{c,t+1}$ with the state variables, x_t, σ_t^2, q_t, and λ_t, and furthermore, substituting this $r_{c,t+1}$ for the Euler equation (3), we can derive an identity in terms of those of the state variables. Solving that identity in the same manner as Bansal and Yaron[2004], Bollerslev, et.al.[2009], Drechsler and Yaron[2011], etc., we can derive the equilibrium solutions for the four parameters as follows:

$$\begin{aligned} A_x &= \frac{\gamma - 1}{\theta(\kappa_1 \rho_x - 1)} \\ A_\sigma &= -\frac{1}{2}\frac{(1-\gamma)^2 \varphi_\eta^2 + \theta^2 \kappa_1^2 A_x^2 \varphi_e^2}{\theta(\kappa_1 \rho_\sigma - 1)} \\ A_\lambda &= \frac{2 - \exp(\frac{1}{2}\theta^2 \kappa_1^2 A_x^2 \delta_x^2) - \exp(\frac{1}{2}\theta^2 \kappa_1^2 A_\sigma^2 \delta_\sigma^2)}{\theta(\kappa_1 \rho_\lambda - 1)} \end{aligned} \tag{11}$$

A_q is a solution of the quadratic equation presented below:

$$\theta A_q(\kappa_1 \rho_q - 1) + \frac{\theta^2 \kappa_1^2}{2}\Big[A_\sigma^2 + A_q^2 \varphi_\xi^2 + 2A_q A_\lambda \varphi_\xi \varphi_u \rho + A_\lambda^2 \varphi_u^2\Big] = 0$$

The following proposition can be easily proved by the expressions of (11):

Proposition 1 *If $\gamma > 1$ and $\psi > 1$, then, $A_x > 0$, $A_\sigma < 0$, $A_q < 0$, and $A_\lambda < 0$.*

The above proposition suggests that if the IES and risk aversion are higher than 1, then a rise in each of the state variables of γ_t^2, q_t, and λ_t lowers the price-consumption ratio.

Having solved for $r_{c,t+1}$ with the four parameters derived above, we can substitute it (and $\triangle c_{t+1} = g_{t+1}$) into m_{t+1} to obtain an expression for the conditional innovation to the log pricing kernel at time $t+1$:

$$\begin{aligned} & m_{t+1} - \mathbb{E}_t[m_{t+1}] \\ &= \theta \log \delta - \frac{\theta}{\psi} \triangle c_{t+1} + (\theta - 1) r_{c,t+1} - \mathbb{E}_t\Big[\theta \log \delta - \frac{\theta}{\psi} \triangle c_{t+1} + (\theta - 1) r_{c,t+1}\Big] \\ &= \Big(-\frac{\theta}{\psi} + \theta - 1\Big)\varphi_\eta \sigma_t \eta_{t+1} + (\theta-1)\kappa_1 A_x \varphi_e \sigma_t e_{t+1} + (\theta - 1)\kappa_1 A_\sigma \sqrt{q_t} w_{t+1} \\ &\quad + (\theta-1)\kappa_1 (A_q \varphi_\xi + A_\lambda \varphi_u \rho)\sqrt{q_t}\xi_{t+1} + (\theta-1)\kappa_1 A_\lambda \varphi_u \sqrt{1-\rho^2}\sqrt{q_t} u_{t+1} \\ &\quad + (\theta-1)\kappa_1 A_x (J_{x,t+1} - \mathbb{E}_t[J_{x,t+1}]) + (\theta-1)\kappa_1 A_\sigma (J_{\sigma^2,t+1} - \mathbb{E}_t[J_{\sigma^2,t+1}]) \\ &= -\Lambda^t (G_t z_{t+1} + J_{t+1} - \mathbb{E}_t[J_{t+1}]), \end{aligned} \tag{12}$$

where

$$
\begin{aligned}
\Lambda &\equiv \begin{pmatrix}\gamma & (1-\theta)\kappa_1 A_x & (1-\theta)\kappa_1 A_\sigma & (1-\theta)\kappa_1 A_q & (1-\theta)\kappa_1 A_\lambda & 0\end{pmatrix}^t \\
G_t &\equiv \begin{pmatrix}
\varphi_\eta \sigma_t & 0 & 0 & 0 & 0 & 0 \\
0 & \varphi_e \sigma_t & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{q_t} & 0 & 0 & 0 \\
0 & 0 & 0 & \varphi_\xi \sqrt{q_t} & 0 & 0 \\
0 & 0 & 0 & \rho\varphi_u \sqrt{q_t} & \varphi_u \sqrt{1-\rho^2}\sqrt{q_t} & 0 \\
0 & 0 & 0 & 0 & 0 & \varphi_\zeta \sigma_t
\end{pmatrix}, \\
z_{t+1} &\equiv \begin{pmatrix}\eta_{t+1} & e_{t+1} & w_{t+1} & \xi_{t+1} & u_{t+1} & \zeta_{t+1}\end{pmatrix}^t, \\
J_{t+1} &\equiv \begin{pmatrix}0 & J_{x,t+1} & J_{\sigma^2,t+1} & 0 & 0 & 0\end{pmatrix}^t, \\
\mathbb{E}_t[J_{t+1}] &\equiv \begin{pmatrix}0 & \mathbb{E}_t[J_{x,t+1}] & \mathbb{E}_t[J_{\sigma^2,t+1}] & 0 & 0 & 0\end{pmatrix}^t.
\end{aligned}
\tag{13}
$$

Thus, Λ can be interpreted as the price of risk for Gaussian shocks and also the sensitivity of the IMRS to the jump shocks. From the expression for Λ, one can see that the prices of risk are determined by the A coefficients, that is, A_x, A_σ, A_q, and A_λ. The expression for Λ also shows that the signs of the risk prices depend on the signs of the A coefficients and $(1-\theta)$. In particular, when $\gamma = \frac{1}{\psi}$, $\theta = 1$, and we are in the case of constant relative risk aversion (CRRA) preferences, it is clear that only the transient shock to consumption $z_{c,t+1}$ is priced, and prices do not separately reflect the risk of shocks to x_t (long-run risk), σ_t^2 (volatility-related risk), q_t (variance-of-variance-related risk), and λ_t (jump intensity-related risk).

In the discussion and calibrations explored below, we especially focus on the case in which the agent's risk aversion γ and the IES ψ are both greater than 1, which implies that $A_x > 0$, $A_\sigma < 0$, $A_q < 0$, and $A_\lambda < 0$ by the proposition presented above. Thus, positive shocks to long-run growth decrease the IMRS, while positive shocks to the levels of the other state variables, σ_t^2, q_t, and λ_t, increase the IMRS. Note that in this case, since $(1-\theta) > 0$, each of the A coefficients has the same sign as the corresponding price of risk.

To study the risk premium in the moments of the market returns, we first need to solve for the market return. A share in the market is modeled as a claim to a dividend with growth process given by $g_{d,t}$. To solve for the price of a market share, we proceed along the same lines as for the consumption claim and solve for $v_{m,t+1}$, the log price-dividend ratio of the market, by using the the conjecture (10), Campbell and Shiller[1988]-approximation (8), and the Euler equation (3) [2] .

[2] Because the datails of the four parameters, $A_{x,m}$, A_{σ_m}, $A_{q,m}$, and $A_{\lambda,m}$, are insignificant and do not affect the discussion explored in the following at all, for simplicity, we express the parameters, $A_{x,m}$, A_{σ_m}, $A_{q,m}$, and $A_{\lambda,m}$, as they are and do not show explicit representations of those parameters in this paper.

With the equilibrium solutions for the parameters of $A_{x,m}$, $A_{\sigma,m}$, $A_{q,m}$, and $A_{\lambda,m}$ in (10), we can obtain an expression for $r_{m,t+1}$ in terms of the state variables and its innovations (by substituting the expression for $v_{m,t(+1)}$ into (8)):

$$\begin{aligned} r_{m,t+1} = {} & \kappa_{0,m} + \kappa_{1,m}A_{0,m} + \kappa_{1,m}A_{\sigma,m}\mu_d + \kappa_{1,m}A_{q,m}\mu_g + \kappa_{1,m}A_{\lambda,m}\mu_\lambda - A_{0,m} + \mu_d \\ & + (\kappa_{1,m}A_{x,m}\rho_x - A_{x,m} + \rho_d)x_t \\ & + (\kappa_{1,m}A_{\sigma,m}\rho_\sigma - A_{\sigma,m}){\sigma_t}^2 \\ & + (\kappa_{1,m}A_{q,m}\rho_q - A_{q,m})q_t \\ & + (\kappa_{1,m}A_{\lambda,m}\rho_\lambda - A_{\lambda,m})\lambda_t \\ & + \kappa_{1,m}A_{x,m}\varphi_e\sigma_t e_{t+1} + \kappa_{1,m}A_{\sigma,m}\sqrt{q_t}w_{t+1} \\ & + \kappa_{1,m}(A_{q,m}\varphi_\xi + A_{\lambda,m}\varphi_u\rho)\sqrt{q_t}\xi_{t+1} \\ & + \kappa_{1,m}A_{\lambda,m}\varphi_u\sqrt{1-\rho^2}\sqrt{q_t}u_{t+1} + \varphi_\zeta\sigma_t\zeta_{t+1} \\ & + \kappa_{1,m}A_{x,m}J_{x,t+1} + \kappa_{1,m}A_{\sigma,m}J_{\sigma^2,t+1} \\ = {} & r_0 + (B_r^t F - A_m^t)Y_t + B_r^t G_t z_{t+1} + B_r^t J_{t+1}, \end{aligned} \tag{14}$$

where

$$r_0 \equiv \kappa_{0,m} + (\kappa_{1,m} - 1)A_{0,m} + (\kappa_{1,m}A_{\sigma,m} + 1)\mu_d + \kappa_{1,m}A_{q,m}\mu_g + \kappa_{1,m}A_{\lambda,m}\mu_\lambda,$$

$$B_r \equiv \kappa_{1,m}A_m + e_d \quad ,$$

$$A_m \equiv \begin{pmatrix} 0 \\ A_{x,m} \\ A_{\sigma,m} \\ A_{q,m} \\ A_{\lambda,m} \\ 0 \end{pmatrix}, \quad e_d \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad F \equiv \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \rho_x & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_\sigma & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_q & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_\lambda & 0 \\ 0 & \rho_d & 0 & 0 & 0 & 0 \end{pmatrix}, \quad Y_t \equiv \begin{pmatrix} g_t \\ x_t \\ {\sigma_t}^2 \\ q_t \\ \lambda_t \\ g_{d,t} \end{pmatrix}. \tag{15}$$

2.3 Risk Premiums in Higher-Order Moments in Equilibrium

Before proceeding to investigating the risk premiums in higher-order moments in equilibrium, we need to add some further explanation on the jump dynamics and the features of the pricing kernel introduced above.

To handle the jumps, we introduce some notation. Let $\psi_k(u_k) = \mathbb{E}[\exp(u_k\epsilon_k)]$, $k = x$ or σ^2 (i.e., ψ_k is the moment-generating function (mgf) of the jump size ϵ_k). The mgf for the jump component of k, $\mathbb{E}[\exp(u_k J_{k,t+1})]$, then equals $\exp(\Psi_{t,k}(u_k))$, where $\Psi_{t,k}(u_k) = \lambda_{k,t}(\psi_k(u_k) - 1)$. $\Psi_{t,k}$ is called the cumulant-generating function (cgf) of $J_{k,t+1}$ and is a very helpful tool for calculating asset pricing moments. The reason is that its n-th derivative evaluated at 0 equals the n-th central moment of $J_{k,t+1}$.

Regarding the features of the pricing kernel, we can show what described below in line with Drechsler and Yaron[2011]. Let us set the Radon-Nikodym derivative $\frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{M_{t+1}}{\mathbb{E}_t[M_{t+1}]}$, where $\mathbb{P}$ is the physical probability measure and $\mathbb{Q}$ is the risk-neutral probability measure in our economy. From (12), we have $\frac{M_{t+1}}{\mathbb{E}_t[M_{t+1}]} \propto \exp(-\Lambda^t(G_t z_{t+1} + J_{t+1}))$. Since z_{t+1} and J_{t+1} are independent, we can treat their measure transformations between $\mathbb{P}$ and $\mathbb{Q}$ separately. As a consequence, Drechsler and Yaron[2011] show that

$$z_{t+1} \overset{\mathbb{Q}}{\sim} N(-G_t'\Lambda, I), \tag{16}$$

where I is the identity matrix in $\mathbb{R}^{6\times 6}$. That is to say that, under $\mathbb{Q}$, z_{t+1} is still a vector of independent normals with unit variances, but with a shift in the mean.

For the case of J_{t+1}, we could also proceed by transforming the probability density function directly. As guided in Drechsler and Yaron[2011], Proposition (9.6) in Cont and Tankov[2004] shows that under $\mathbb{Q}$, the $J_{t+1,k}$ are still compound Poisson processes, but with cgf given by

$$\Psi^{\mathbb{Q}}_{t,k}(u_k) = \lambda_{k,t}\psi_k(-\Lambda_k)\Big(\frac{\psi_k(u_k - \Lambda_k)}{\psi_k(-\Lambda_k)} - 1\Big), \tag{17}$$

where $k = x$ or $k = \sigma^2$ and Λ_x denotes the price of risk for the LRR-factor x_t, that is, $(1-\theta)\kappa_1 A_x$, and Λ_{σ^2} denotes the price of risk for the variance of consumption growth rate, that is, $(1-\theta)\kappa_1 A_\sigma$. (See (13)) In the following discussion, we use the stylized facts mentioned above to calculate the higher-order moments of the market returns and to investigate the risk premiums in those of moments.

2.3.1 The Variance Risk Premium in Equilibrium

According to Bollerslev, et.al.[2009] and Drechsler and Yaron[2011], the variance risk premium in equilibrium, vp_t, is defined by

$$vp_t \equiv \mathbb{E}^{\mathbb{Q}}_t[\mathrm{Var}^{\mathbb{Q}}_{t+1}(r_{m,t+2})] - \mathbb{E}^{\mathbb{P}}_t[\mathrm{Var}^{\mathbb{P}}_{t+1}(r_{m,t+2})], \tag{18}$$

where $Var^{\mathbb{P}}_{t+1}$ ($Var^{\mathbb{Q}}_{t+1}$) is the variance operator under the physical (risk-neutral) probability measure. From (14), the conditional variance of the market return $r_{m,t+2}$ on time $t+1$ under $\mathbb{P}$ can be obtained easily as follows:

$$\begin{aligned}\mathrm{Var}^{\mathbb{P}}_{t+1}(r_{m,t+2}) &= B_r^t G_{t+1} G_{t+1}^t B_r + \sum_i B_r^2(i)\mathrm{Var}^{\mathbb{P}}_{t+1}(J_{i,t+2}) \\ &= B_r^t G_{t+1} G_{t+1}^t B_r + B_r^{2^t}\Psi^{(2)}_{t+1}(0),\end{aligned} \tag{19}$$

where

$$\begin{aligned} B_r &= \kappa_{1,m}A_m + e_d \quad (\because (15)) \\ &\equiv \Big(B_r(1) \quad B_r(2) \quad B_r(3) \quad B_r(4) \quad B_r(5) \quad B_r(6)\Big)^t \in \mathbb{R}^6, \\ B_r^2 &\equiv \Big(B_r^2(1) \quad B_r^2(2) \quad B_r^2(3) \quad B_r^2(4) \quad B_r^2(5) \quad B_r^2(6)\Big)^t \in \mathbb{R}^6, \\ \Psi^{(2)}_{t+1}(0) &\equiv \Big(0 \quad \Psi^{(2)}_{t+1,x}(0) \quad \Psi^{(2)}_{t+1,\sigma^2}(0) \quad 0 \quad 0 \quad 0\Big)^t \in \mathbb{R}^6, \end{aligned}$$

and $\Psi^{(2)}_{t+1,x}(0)$ and $\Psi^{(2)}_{t+1,\sigma^2}(0)$ are respectively the second derivative of the cgf (cumulant-generating function) for $J_{x,t+1}$ and $J_{\sigma^2,t+1}$ evaluated at 0, that is,

$$\Psi^{(2)}_{t+1,x}(0) \equiv \frac{\partial^2}{\partial u^2}\Psi_{t+1,x}(u) \mid_{u=0} = \frac{\partial^2}{\partial u^2}\lambda_{x,t+1}(\psi_x(u)-1) \mid_{u=0},$$
$$\Psi^{(2)}_{t+1,\sigma}(0) \equiv \frac{\partial^2}{\partial u^2}\Psi_{t+1,\sigma^2}(u) \mid_{u=0} = \frac{\partial^2}{\partial u^2}\lambda_{\sigma^2,t+1}(\psi_{\sigma^2}(u)-1) \mid_{u=0}.$$

Thus the expression of (19) is rearranged to the following representation,

$$\begin{aligned}\mathrm{Var}^{\mathbb{P}}_{t+1}(r_{m,t+2}) &= B_r^t G_{t+1} G_{t+1}^t B_r + B_r^{2t}\Psi^{(2)}_{t+1}(0) \\ &= B_r^t(H_{\sigma^2}\sigma^2_{t+1} + H_q q_{t+1})B_r + B_r^{2t}\mathrm{diag}\Big(\psi^{(2)}(0)\Big)\Pi_{t+1},\end{aligned} \tag{20}$$

where

$$H_{\sigma^2} \equiv \begin{pmatrix} \varphi_\eta^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \varphi_e^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \varphi_\zeta^2 \end{pmatrix}, \quad H_q \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \varphi_\xi^2 & \rho\varphi_\xi\varphi_u & 0 \\ 0 & 0 & 0 & \rho\varphi_\xi\varphi_u & \varphi_u^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathrm{diag}\Big(\psi^{(2)}(0)\Big) \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_x^{(2)}(0) & 0 & 0 & 0 & 0 \\ 0 & 0 & \psi_{\sigma^2}^{(2)}(0) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Pi_{t+1} \equiv \begin{pmatrix} 0 \\ \lambda_{x,t+1} \\ \lambda_{\sigma^2,t+1} \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Under the risk-neutral probability measure $\mathbb{Q}$, the conditional variance of the market return $r_{m,t+2}$ on time $t+1$ also can be obtained in the same manner demonstrated above. As a consequence, we can show the following proposition with the definition of the variance risk premium (18).

Proposition 2 (The Variance Risk Premium in Equilibrium) *In equilibrium, the variance risk premium at time t, vp_t, is linear to the variance-of-variance, q_t, and the jump intensity, λ_t, and the explicit representation of it is provided as follows:*

$$vp_t = \beta_{vp,c} + \beta_{vp,q}q_t + \beta_{vp,\lambda}\lambda_t,$$

where

$$\begin{aligned}\beta_{vp,c} &\equiv \Big[l_x B_r^2(2)(\psi_x^{(2)}(-\Lambda_x) - \psi_x^{(2)}(0)) + l_{\sigma^2}B_r^2(3)(\psi_{\sigma^2}^{(2)}(-\Lambda_{\sigma^2}) - \psi_{\sigma^2}^{(2)}(0))\Big]\mu_\lambda, \\ \beta_{vp,q} &\equiv -B_r^t\Big[\Lambda_{\sigma^2}H_{\sigma^2} + \varphi_\xi(\varphi_\xi\Lambda_q + \rho\varphi_u\Lambda_\lambda)H_q\Big]B_r \\ &\quad - \varphi_u(\rho\varphi_\xi\Lambda_q + \varphi_u\Lambda_\lambda)(l_xB_r^2(2)\psi_x^{(2)}(-\Lambda_x) + l_{\sigma^2}B_r^2(3)\psi_{\sigma^2}^{(2)}(-\Lambda_{\sigma^2})), \\ \beta_{vp,\lambda} &\equiv B_r^tH_{\sigma^2}B_r\psi_{\sigma^2}^{(1)}(-\Lambda_{\sigma^2}) \\ &\quad + \Big[l_xB_r^2(2)(\psi_x^{(2)}(-\Lambda_x) - \psi_x^{(2)}(0)) + l_{\sigma^2}B_r^2(3)(\psi_{\sigma^2}^{(2)}(-\Lambda_{\sigma^2}) - \psi_{\sigma^2}^{(2)}(0))\Big]\rho_\lambda.\end{aligned} \tag{21}$$

Proof *See the Appendix.*

A number of interesting implications arise from the expression (21). In particular, any temporal variation in the endogenously generated variance risk premium is solely due to the variance-of-variance q_t and the jump intensity λ_t. Moreover, provided that $\theta < 0$, $\Lambda_x > 0$, and $\Lambda_{\sigma^2} < 0$, as would be implied by $\gamma > 1$ and $\psi > 1$, the factor loading to the jump intensity, that is, $\beta_{vp,\lambda}$, is guaranteed to be positive, but that to the variance-of-variance, that is, $\beta_{vp,q}$, can be both positive and negative in general. However, if the correlation between the dynamics of the variance-of-variance and that of the jump intensity, that is , ρ, is posituve, then $\beta_{vp,q}$ is also guaranteed to be positive due to the facts that $\Lambda_q < 0$ and $\Lambda_\lambda < 0$.

2.3.2 The Skewness Risk Premium in Equilibrium

Based on the same manner used to derive the expression (20) in the previous subsection, we can also derive the explicit representations for the skewness of the market returns under $\mathbb{P}$ and $\mathbb{Q}$, respectively:

$$\begin{aligned} \mathrm{Skew}_t^{\mathbb{P}}(r_{m,t+1}) &= B_r^{3^t} \mathrm{diag}(\psi^{(3)}(0))\Pi_{\mathrm{t}}, \\ \mathrm{Skew}_t^{\mathbb{Q}}(r_{m,t+1}) &= B_r^{3^t} \mathrm{diag}(\psi^{(3)}(-\Lambda))\Pi_{\mathrm{t}}, \end{aligned} \tag{22}$$

where

$$B_r^3 \equiv \begin{pmatrix} B_r^3(1) & B_r^3(2) & B_r^3(3) & B_r^3(4) & B_r^3(5) & B_r^3(6) \end{pmatrix}^t,$$

$$\mathrm{diag}\Big(\psi^{(3)}(0)\Big) \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_x^{(3)}(0) & 0 & 0 & 0 & 0 \\ 0 & 0 & \psi_{\sigma^2}^{(3)}(0) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathrm{diag}\Big(\psi^{(3)}(-\Lambda)\Big) \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_x^{(3)}(-\Lambda_x) & 0 & 0 & 0 & 0 \\ 0 & 0 & \psi_{\sigma^2}^{(3)}(-\Lambda_{\sigma^2}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

In this paper, we define the skewness risk premium in equilibrium at time t, skp_t, as the following expression:

$$skp_t \equiv \mathbb{E}_t^{\mathbb{Q}}[\mathrm{Skew}_{t+1}^{\mathbb{Q}}(r_{m,t+2})] - \mathbb{E}_t^{\mathbb{P}}[\mathrm{Skew}_{t+1}^{\mathbb{P}}(r_{m,t+2})]. \tag{23}$$

Substituting (22) into (23), the explicit representation for the skewness risk premium can be obtained as the following:

$$skp_t \equiv B_r^{3t}\mathrm{diag}(\psi^{(3)}(-\Lambda))\mathbb{E}_t^{\mathbb{Q}}[\Pi_{t+1}] - B_r^{3t}\mathrm{diag}(\psi^{(3)}(0))\mathbb{E}_t^{\mathbb{P}}[\Pi_{t+1}]. \tag{24}$$

We also find a number of interesting implications from the expressions of (22) and (24). First, in the case that there is no jump to fundamentals in the economy, that is, in the case of $\Pi_t \equiv 0$ $(\in \mathbb{R}^6)$, it is clear that the skewness of the market returns should be zero due to (22). Thus, the existence of the nonzero skewness of the market returns crucially depend on the existence of the jumps to fundamentals in the economy. Second, any temporal variation in the endogenously generated skewness and skewness risk premium are solely due to the jump intensity process λ_t. For example, if the jump intensity is constant, then it is clear that the skewness (under $\mathbb{P}$ and $\mathbb{Q}$) and skewness risk premium should be constant by (22) and (24). Third, since we have the fact of $A_\sigma < 0$ by the proposition 1, then in the case that the jump to the variance of consumption growth rate exists, that is, in the case that $\lambda_{\sigma^2,t} > 0$, we can show easily via (22) that the risk-neutral skewness at time t, $\mathrm{Skew}_t^{\mathbb{Q}}(r_{m,t+1})$, should be negative. Finally, we can also find via (24) that in the case that either $\lambda_{x,t} > 0$ or $\lambda_{\sigma^2,t} > 0$ is satisfied, the skewness risk premium at time t, skp_t, in equilibrium also should be negative due to the facts of $A_x > 0$ and $A_\sigma < 0$.

Based on the definition of (23), let us provide the proposition on the representation for the skewness risk premium in equilibrium.

Proposition 3 (The Skewness Risk Premium in Equilibrium) *In equilibrium, the skewness risk premium at time t, skp_t, is linear to the variance-of-variance, q_t, and the jump intensity, λ_t, and the explicit representation of it is provided as follows:*

$$\begin{aligned}
& skp_t = \beta_{sp,c} + \beta_{sp,q} q_t + \beta_{sp,\lambda}\lambda_t \\
& where \\
& \beta_{sp,c} \equiv \left[l_x B_r^3(2)\psi_x{}^{(3)}(-\Lambda_x) + l_{\sigma^2}B_r^3(3)\psi_{\sigma^2}{}^{(3)}(-\Lambda_{\sigma^2})\right]\mu_\lambda \\
& \beta_{sp,q} \equiv \left[l_x B_r^3(2)\psi_x{}^{(3)}(-\Lambda_x) + l_{\sigma^2}B_r^3(3)\psi_{\sigma^2}{}^{(3)}(-\Lambda_{\sigma^2})\right]\varphi_u(-\rho\varphi_\xi\Lambda_q - \varphi_u\Lambda_\lambda) \\
& \beta_{sp,\lambda} \equiv \left[l_x B_r^3(2)\psi_x{}^{(3)}(-\Lambda_x) + l_{\sigma^2}B_r^3(3)\psi_{\sigma^2}{}^{(3)}(-\Lambda_{\sigma^2})\right]\rho_\lambda
\end{aligned} \tag{25}$$

Proof *Considering (6), (16), and the definition of the moment-generating function, it is trivial to derive the above expression.* □

From the above proposion, we find that any temporal variation in the endogenously generated skewness risk premium is also solely due to the variance-of-variance q_t and the jump intensity λ_t as well as the volatility risk premium. Moreover, provided that $\Lambda_x > 0$ and $\Lambda_{\sigma^2} < 0$, the factor loading to the jump intensity, that is, $\beta_{sp,\lambda}$, is guaranteed to be negative, but that to the variance-of-variance, that is, $\beta_{sp,q}$, can be both positive and negative in general. However, if the correlation between the dynamics of the variance-of-variance and that of the jump intensity, that is , ρ, is posituve, then $\beta_{sp,q}$ is also guaranteed to be negative due to the facts that $\Lambda_q < 0$ and $\Lambda_\lambda < 0$.

Before we turn to the next discussion, it will be useful to mention about some features of the higher-order moments of the market returns and the risk premiums in them.

First, as mentioned in the introduction in this paper, the usual assumption of an affine structure on the jump intensity process λ_t, that is, $\lambda_t = l_0 + l_1\sigma_t^2$ where $l_0, l_1 > 0$ and σ_t^2 is the variance of consumption growth rate, in the previous studies such as Drechsler and Yaron[2011] can not explain an empirical fact on a simultaneous relation between monthly stock returns and monthly changes of the option-implied skewness shown by (1). It is because, under such assumption, we can show analytically that the regression parameters to $\triangle ISkew_{t+1}$ in (1) should be positive. However, based on our model provided above, the correlation between the one-step-ahead market return, $r_{m,t+1}$, and the one-step-ahead change of risk-neutral skewness, $\triangle Skew^{\mathbb{Q}}_{t+1} \equiv Skew^{\mathbb{Q}}_{t+1} - Skew^{\mathbb{Q}}_t$, at time t can be derived with (14) and (22) as follows:

$$\begin{aligned}\mathrm{Corr}\Big(r_{m,t+1}, \triangle Skew^{\mathbb{Q}}_{t+1}\Big) &= K\varphi_u\kappa_{1,m}(\rho A_{q,m} + \varphi_u A_{\lambda,m})q_t,\\ \text{where}\quad K &\equiv l_x B_r^3(2){\psi_x}^{(3)}(-\Lambda_x) + l_{\sigma^2}B_r^3(3){\psi_{\sigma^2}}^{(3)}(-\Lambda_{\sigma^2}).\end{aligned}$$

According to the above expression, we find that when $\rho < -\varphi_u\frac{A_{\lambda,m}}{A_{q,m}}$ the correlation between $r_{m,t+1}$ and $\triangle Skew^{\mathbb{Q}}_{t+1} \equiv Skew^{\mathbb{Q}}_{t+1} - Skew^{\mathbb{Q}}_t$ should be negative because, in the case of $\gamma > 1$ and $\psi > 1$, it is proved that K is negative . This observation is consistent with the empirical fact of (1) shown in the introduction of this paper. Thus, we would like to emphasize that there is considerable validity in our model setting compared with the previous studies such as Drechsler and Yaron[2011], etc.

Second, although both the variance risk premium and the skewness risk premium are linear to the variance-of-variance q_t and the jump intensity λ_t, we can show that they are mutually independent because of the fact that $det \equiv \beta_{vp,q}\beta_{sp,\lambda} - \beta_{vp,\lambda}\beta_{sp,q}$ is not zero, which will be proved in the following section.

2.4 An Equity Risk Premium Representation

In this subsection, let us show an equity risk premium representation with the variance and skewness risk premiums in equilibrium. In the beginning, we start with an expression for the equity risk premium provided by Drechsler and Yaron[2011] as follows:

$$\log \mathbb{E}_t(R_{m,t+1}) - r_{f,t} = B_r^t G_t G_t^t \Lambda + \Pi_t^t(\psi(B_r) - 1 - \psi(B_r - \Lambda) + \psi(-\Lambda)),$$

where

$$\psi(B_r) \equiv \begin{pmatrix} 0 \\ \psi_x(B_r(2)) \\ \psi_{\sigma^2}(B_r(3)) \\ 0 \\ 0 \\ 0 \end{pmatrix}, \psi(B_r - \Lambda) \equiv \begin{pmatrix} 0 \\ \psi_x(B_r(2) - \Lambda_x) \\ \psi_{\sigma^2}(B_r(3) - \Lambda_{\sigma^2}) \\ 0 \\ 0 \\ 0 \end{pmatrix}, \psi(-\Lambda) \equiv \begin{pmatrix} 0 \\ \psi_x(-\Lambda_x) \\ \psi_{\sigma^2}(-\Lambda_{\sigma^2}) \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

As mentioned in Drechsler and Yaron[2011], the first term, $B_r^t G_t G_t^t \Lambda$, represents the contributions of the Gaussian shocks to the equity risk premium. In particular, according to the expression of $G_t G_t^t = H_{\sigma^2}\sigma_t^2 + H_q q_t$ (see (20)), this term aggregates both the risk-return tradeoff relationship and a true premium for variance risk. The next terms, $\Pi_t^t(\psi(B_r) - 1) - \Pi_t^t(\psi(B_r - \Lambda) - \psi(-\Lambda))$, represent the contributions from the jump processes. The derivation of this expression is presented in Appendix in Drechsler and Yaron[2011].

The $r_{f,t}$ is the risk-free rate at time t in the economy and the explicit expression of this $r_{f,t}$ is provided in the Appendix.

With the expression of $G_t G_t^t = H_{\sigma^2}\sigma_t^2 + H_q q_t$ and $\Pi_t \equiv \begin{pmatrix} 0 & \lambda_{x,t} & \lambda_{\sigma^2,t} & 0 & 0 & 0 \end{pmatrix}^t$, the following representation can be obtained via the expression for the equity risk premium shown above.

$$\begin{aligned} \log \mathbb{E}_t(R_{m,t+1}) - r_{f,t} &= \beta_{er,\sigma}\sigma_t^2 + \beta_{er,q} q_t + \beta_{er,\lambda}\lambda_t, \\ \textit{where} \quad & \beta_{er,\sigma} \equiv B_r^t H_\sigma \Lambda, \\ & \beta_{er,q} \equiv B_r^t H_q \Lambda, \\ & \beta_{er,\lambda} \equiv l_x \Big[\psi_x(B_r(2)) - 1 - \psi_x(B_r(2) - \Lambda_x) + \psi_x(-\Lambda_x)\Big] \\ & \qquad + l_\sigma \Big[\psi_{\sigma^2}(B_r(3)) - 1 - \psi_\sigma(B_r(3) - \Lambda_{\sigma^2}) + \psi_\sigma(-\Lambda_{\sigma^2})\Big]. \end{aligned} \tag{26}$$

As shown in (26), the equity risk premium is driven by the state variables of σ_t^2, q_t, and λ_t and have a time-varying nature essentially because those of variables have the stochastic nature. In particular, in the case of $\gamma > 1$ and $\psi > 1$, it is proved that $\beta_{er,\sigma} > 0$, $\beta_{er,q} > 0$, and $\beta_{er,\lambda} > 0$ because of the facts of $\Lambda_x > 0$, $\Lambda_{\sigma^2} < 0$, $\Lambda_q < 0$, and $\Lambda_\lambda < 0$ from Proposition 1, so that if each of the state variables increases, then the equity risk premium also increases, and vice versa.

The conditional variance of the equity return at time t is also presented by

$$\begin{aligned} \mathrm{Var}_t^{\mathbb{P}}(r_{m,t+1}) &= B_r^t G_t G_t^t B_r + B_r^{2t} \mathrm{diag}\Big(\psi^{(2)}(0)\Big)\Pi_\mathrm{t} \\ &= B_r^t H_{\sigma^2} B_r \sigma_t^2 + B_r^t H_q B_r q_t + (l_x B_r^2(2)\psi_x^{(2)}(0) + l_{\sigma^2} B_r^2(3)\psi_{\sigma^2}^{(2)}(0))\lambda_t \\ &\equiv \beta_{var,\sigma}\sigma_t^2 + \beta_{var,q} q_t + \beta_{var,\lambda}\lambda_t, \end{aligned}$$

so that with (21), (25), (26), and the above expression for the conditional variance of the equity return we can derive an explicit equity risk premium representation of a linear factor pricing model with the variance and skewness risk premiums and the conditional variance of the equity return.

Proposition 4 (An Explicit Representation for the Equity Risk Premium)

$$\log \mathbb{E}_t(R_{m,t+1}) - r_{f,t} = \pi_c + \pi_{var}\mathrm{Var}_t^{\mathbb{P}}(r_{m,t+1}) + \pi_{vp}vp_t + \pi_{sp}skp_t,$$

where

$$\begin{aligned}
\pi_c &\equiv \Big(-\frac{\beta_{er,\sigma}\beta_{var,q}}{\beta_{var,\sigma}} + \beta_{er,q}\Big)\frac{-\beta_{sp,\lambda}\beta_{vp,c} + \beta_{vp,\lambda}\beta_{sp,c}}{det} \\
&\quad + \Big(-\frac{\beta_{er,\sigma}\beta_{var,\lambda}}{\beta_{var,\sigma}} + \beta_{er,\lambda}\Big)\frac{\beta_{sp,q}\beta_{vp,c} - \beta_{vp,q}\beta_{sp,c}}{det}, \\
\pi_{var} &\equiv \frac{\beta_{er,\sigma}}{\beta_{var,\sigma}}, \\
\pi_{vp} &\equiv \Big(-\frac{\beta_{er,\sigma}\beta_{var,q}}{\beta_{var,\sigma}} + \beta_{er,q}\Big)\frac{\beta sp,\lambda}{det} - \Big(-\frac{\beta_{er,\sigma}\beta_{var,\lambda}}{\beta_{var,\sigma}} + \beta_{er,\lambda}\Big)\frac{\beta sp,q}{det}, \\
\pi_{sp} &\equiv -\Big(-\frac{\beta_{er,\sigma}\beta_{var,q}}{\beta_{var,\sigma}} + \beta_{er,q}\Big)\frac{\beta vp,\lambda}{det} + \Big(-\frac{\beta_{er,\sigma}\beta_{var,\lambda}}{\beta_{var,\sigma}} + \beta_{er,\lambda}\Big)\frac{\beta vp,q}{det}, \\
det &\equiv \beta_{vp,q}\beta_{sp,\lambda} - \beta_{vp,\lambda}\beta_{sp,q}.
\end{aligned} \tag{27}$$

This representation of (27) suggests that the skewness risk premium, as well as the variance risk premium and the conditional variance of the market return, constitutes the dominant source of the variation in the equity risk premium. In the following section, we can show that *det* in (27) is not zero under the usual parameter condition, so that the skewness risk premium has an essential source of the variation in the equity risk premium, which is different from that of the variance risk premium (see Fig.1).

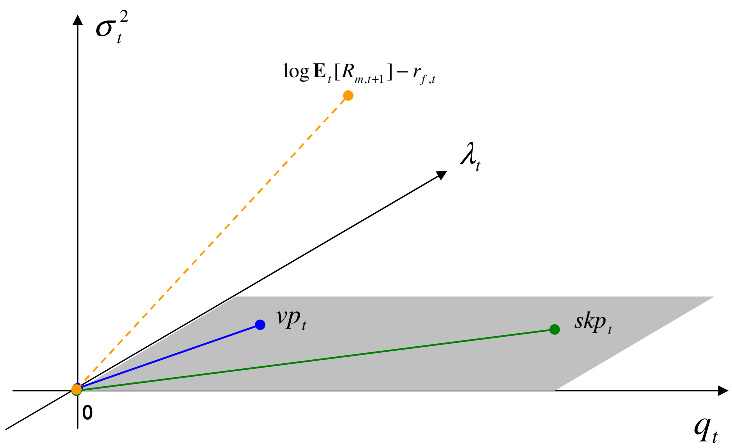

Fig. 1: The Risk Premiums in Higher-Order Moments and the Equity Risk Premium

Some recent studies such as Bali and Hovakimian[2009], Yan[2009], Chang, et.al.[2012], Driessen, et.al.[2012], and Rehman and Vilkov[2012] focus on a significant relationship between skewness or jump risks and expected stock returns, and they provide empirical evidence for a significantly positive link between the expected stock returns and the jump or skewness risks. To the best of our knowledge, this result of (27), which suggests an explicit relationship between the skewness risk premium and the expected equity excess return, is the first to provide a theoretical implication in their empirical evidence in terms of the LRR model approach pioneered by Bansal and Yaron[2004].

3 Model Implications

Before turning to an empirical analysis based on the representation of (27), we show the implications from a calibrated version of the theoretical model (27) to help guide and interpret our subsequent empirical reduced form predictability regressions.

Table 1: The Set of Model Parameters

Parameter	Source	(Calibrated) Values
(1) Preference		
ψ	BST	2.5
(2) Consumption Growth		
φ_η	BY	1.0
(3) Long Run Risk		
ρ_x	BY	0.979
φ_e	BY	0.044
(4) Variance		
ρ_σ	BTZ	0.978
(5) Variance-of-Variance		
ρ_q	BTZ	0.8
φ_ξ	BTZ	0.001
(6) Campbell-Shiller		
κ_1	BTZ	0.9
(7) Jump Intensity		
ρ_λ	CM	0.9
φ_u	-	0.01
$\delta_x, \delta_{\sigma^2}$	-	0.01
μ_λ	-	1.0
l_x, l_{σ^2}	-	1.0

This table reports the parameter values used in the calibration of the factor loadings in the theoretical model (27). CM, BY, BTZ, and BST in this table denote values taken directly from Chan and Maheu[2002], Bansal and Yaron[2004], Bollerslev, et.al.[2009], and Bollerslev, et.al.[2012], respectively.

Table 1 reports the parameter values used in the calibration of the factor loadings in the theoretical model (27). CM, BY, BTZ, and BST in this table denote values taken directly from Chan and Maheu[2002], Bansal and Yaron[2004], Bollerslev, et.al.[2009], and Bollerslev, et.al.[2012], respectively. Those of previous studies refer to the unit time interval in the calibrated equilibrium models as a month, and we also refer to that as the same. Based on the parameters exhibited in Table 1, we calibrate the factor loadings of the representation (21) for the variance risk premium and of the representation (25) for the skewness risk premium in equilibrium.

The figures from Fig.2 to Fig.5 show those of factor loadings, $\beta_{vp,q}$, $\beta_{vp,\lambda}$, $\beta_{sp,q}$, and $\beta_{sp,\lambda}$, corresponding to the parameters of the risk aversion parameters γ and the correlation ρ between the volatility of volatility q_t and the jump intensity λ_t. As is shown in the previous section, $\beta_{vp,\lambda}$, which is the factor loading to the jump intensity λ_t in the variance risk premium representation (21), is positive essentially, and under the parameter values exhibited in Table 1, $\beta_{vp,q}$, which is the factor loading to the variance-of-variance q_t in (21), also seems to be positive. These results indicate that when the variance-of-variance and (or) the jump intensity rise(s), the level of the variance risk premium also increases. In contrast, $\beta_{sp,\lambda}$, which is the factor loading to the jump intensity in the skewness risk premium representation (25), is negative essentially as shown in the previous section, and interestingly, $\beta_{sp,q}$, which is the factor loading to the variance-of-variance in (25), can be both positive and negative corresponding to the parameters of γ and ρ. These results on the $\beta_{sp,\lambda}$ and the $\beta_{sp,q}$ indicate that although an increase in the jump intensity reduces the level of the skewness risk premium essentially, but an increase in the variance-of-variance will raise or reduce the level of that risk premium corresponding to the γ and the ρ.

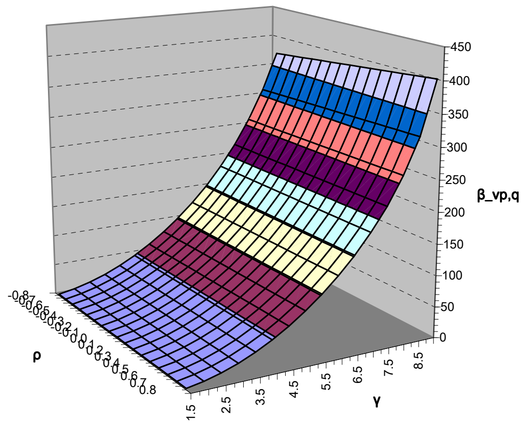

Fig. 2: **The Factor Loading** $\beta_{vp,q}$

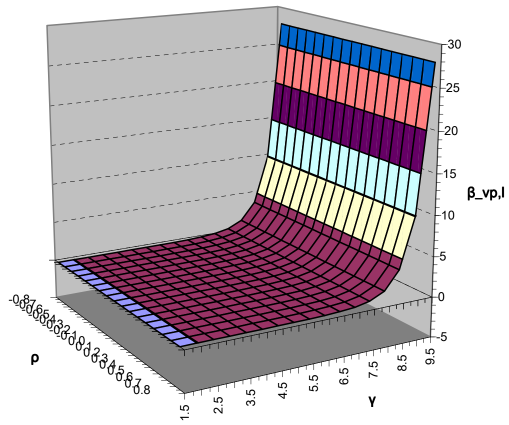

Fig. 3: **The Factor Loading** $\beta_{vp,\lambda}$

The figures from Fig.6 to Fig.8 show the factor loadings to the variance of the market return, the variance risk premium, and the skewness risk premium in the equity risk premium representation (27). It is interesting that both of the π_{Var} and π_{vp} are positive and these results are irrelevant to the values of the γ and the ρ. Those of results are consistent with the previous studies such as Bollerslev, et.al.[2009] and Drechsler and Yaron[2012]. An important point to emphasize is that the factor loading of π_{skp}, which is the loading to the skewness risk premium in (27), can be positive corresponding to the

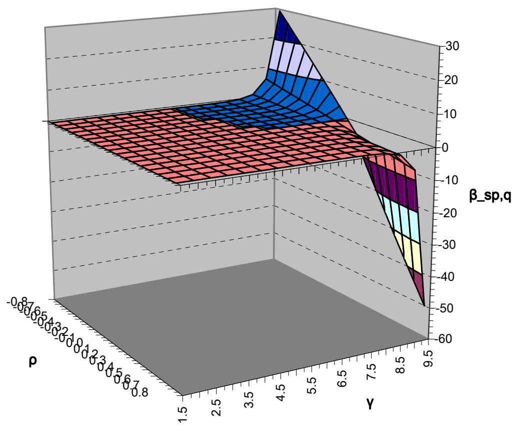

Fig. 4: **The Factor Loading** $\beta_{sp,q}$

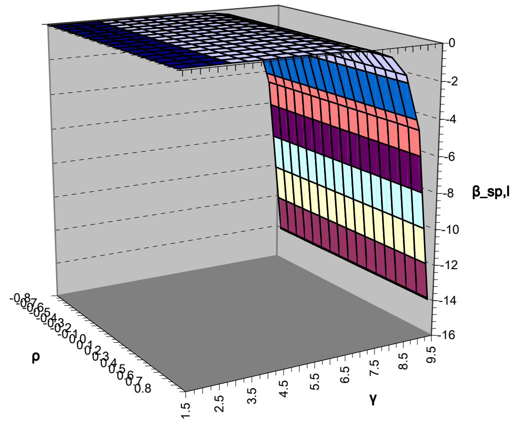

Fig. 5: **The Factor Loading** $\beta_{sp,\lambda}$

risk aversion parameter γ. In particular, when the γ is over 4, it is clear from Fig.8 that the π_{skp} is strictly positive. This result indicates that a decrease in the skewness risk premium, which is the case that the risk-neutral skewness is going to be much smaller than the skewness under the physical measure, reduces the equity risk premium when the γ is over 4. This implication is interesting as it shows the essential contribution of the skewness risk premium to the equity risk premium explicitly implying the sign of the π_{skp} corresponding to the γ and the ρ. There has been no study that tried to provide the theoretical equilibrium model which accounts for a close relationship between the skewness risk premium and the equity risk premium endogeneously. To the best of our knowledge, this is the first paper that demonstrates what mentioned above with a calibration result of the proposed model.

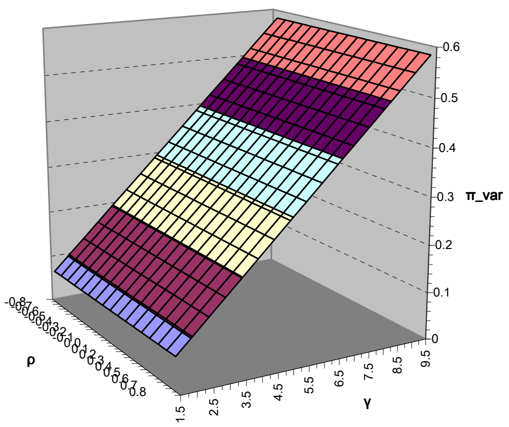

Fig. 6: **The Factor Loading to** $Var_t^{\mathbb{P}}$**:**π_{Var}

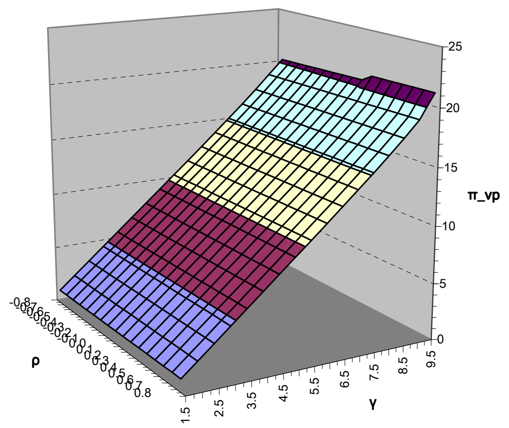

Fig. 7: **The Factor Loading to** vp_t**:**π_{vp}

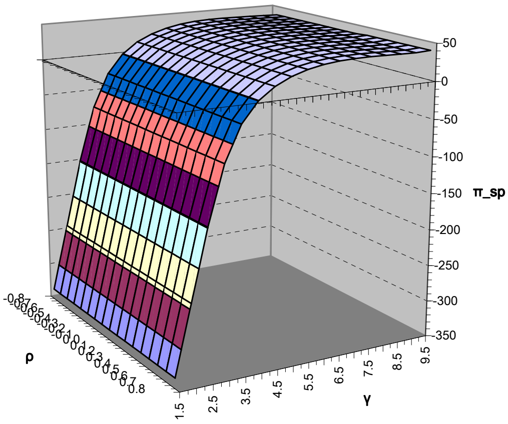

Fig. 8: **The Factor Loading to skp_t:π_{skp}**

4 Empirical Measurements

The theoretical model outlined in the previous section suggests that the variance and skewness risk premiums, as well as the variance of the market return, may serve as a useful predictor for the future market returns. To examine that suggestion empirically, we plan for running some statistical tests based on simple linear regressions of the S & P500 excess returns on different sets of lagged predictor variables including the variance and the skewness risk premiums. We always rely on monthly and quarterly observations. We focus our discussion on the estimated slope coefficients and their statistical significance as determined by the t-statistics. We also report the forecasts' accuracy of the regressions as measured by the corresponding adjusted R^2s.

Before showing the results of the predictive regressions of the S & P500 excess returns, let us note some key points on the measurements for the variance and skewness risk premiums and describe the data used in our analysis explored in the following.

4.1 Measurements for the Higher-Order Moments

Our method for measuring the risk premiums in higher-order moments is similar to that in Bollerslev, et.al.[2009] and Drechsler and Yaron[2011]. As mentioned above, we formally define the variance risk premium as the difference between the risk-neutral and physical expectations of the variance of the market return and also define the skewness risk premium in the same manner. We focus on the one-month- and three-month-forward predictability of those of the risk premiums and use the squared VIX and the SKEW index from the Chicago Board of Options Exchange (CBOE) as our measures for the risk-neutral expected variance and skewness, respectively. The VIX is calculated by the CBOE using the model-free approach to measure 30-day expected volatility of the S

& P500 return. The components of the VIX are near- and next-term put and call options, usually in the first and second SPX (S & P500 index) contract months. The model-free approach used to calculate the VIX is provided by, for example, Demeterfi, et.al.[1999]. The SKEW index from the CBOE is also calculated from the S & P 500 option prices based on the method similar to that used to calculate the VIX, which is obtained by a portfolio of S & P 500 index options that mimics an exposure to the skewness payoff of one-step-ahead cumulative return distribution of the index. The Skew index is also calculated by the model-free approach provided, for example, Bakshi, et.al.[2003]. [3]

For the measures for the expected variance and skewness under the physical measure, we use the current variance and skewness of the S & P500 index return, which are respectively defined as the historical 22 days actual variance estimated based on daily return data of the index and the historical 12 months actual skewness estimated based on monthly return data of the index. To match the definition of those of historical moments of the index return distribution with the risk-neutral expected moments mentioned above, we use the annualized current variance, while the current skewness, which is estimated based on historical 12 months monthly return data, is used as it is. Bollerslev, et.al.[2009] suggest that, for highly persistent variance dynamics, or $\rho_\sigma \approx 1$, the objective expected future variance will obviously be close to the value of the current variance so that the same qualitative implications hold true for the variance difference obtained by replacing $\mathbb{E}_t^{\mathbb{P}}[Var_{t+1}^{\mathbb{P}}(r_{m,t+2})]$ in Equation (18) with the current variance. In a similar point of view, the same may be said of the objective expected future skewness. Moreover, compared to the variance and skewness risk premiums defined by (18) and (23), respectively, the usage of the historical return moments in order to substitute for the objective expected future moments has the advantage that those of risk premiums are directly observable at time t. This is obviously important from a forecasting perspective. It is for these reasons mentioned above that we use the current variance and skewness of the index return to measure the expected variance and skewness under the physical measure.

4.2 Data Description

Our data series for the VIX, SKEW index, and expected variance and skewness under $\mathbb{P}$ covers the period from January 1990 to August 2012. The main limitation on the length of our sample comes from the VIX and SKEW index , since the time series published by the CBOE begins in January 1990. As mentioned in the previous subsection, we rely on the monthly and quarterly data for the squared VIX and SKEW index for quantifying $\mathbb{E}_t^{\mathbb{Q}}[Var_{t+1}^{\mathbb{Q}}(r_{m,t+2})]$ in (18) and $\mathbb{E}_t^{\mathbb{Q}}[Skew_{t+1}^{\mathbb{Q}}(r_{m,t+2})]$ in (23), respectively, and purposely rely on the readily available squared VIX as our measure for that risk-neutral expected

[3] According to the description of the CBOE's SKEW index, we have the proxy for the risk-neutral expected skewness, $\mathbb{E}_t^{\mathbb{Q}}[Skew_{t+1}^{\mathbb{Q}}(r_{m,t+2})]$, as $\frac{1}{10}(100 - SKEW\ \ index)$.

variance and the value of $\frac{1}{10}(100 - SKEW\ \ index)$ as our measure for that risk-neutral expected skewness. The expected variance $\mathbb{E}_t^{\mathbb{P}}[Var_{t+1}^{\mathbb{P}}(r_{m,t+2})]$ and the expected skewness $\mathbb{E}_t^{\mathbb{P}}[Skew_{t+1}^{\mathbb{P}}(r_{m,t+2})]$ at time t are respectively calculated based on the historical index returns as described in the previous subsection.

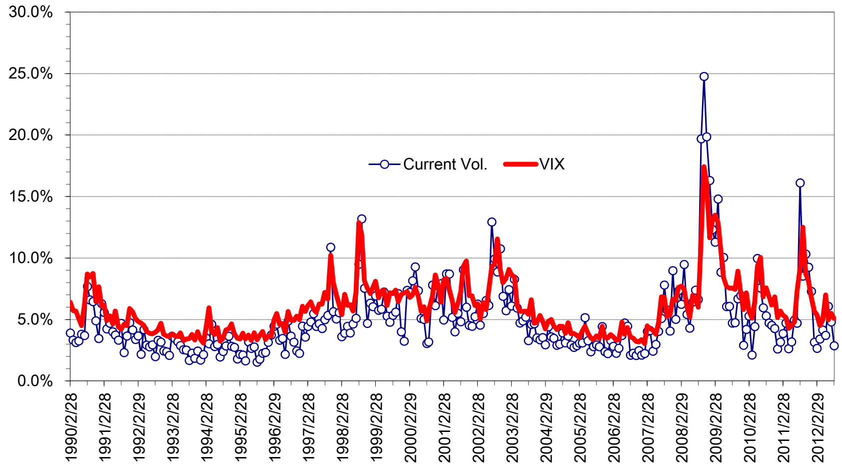

Fig. 9: **The VIX and The Current Volatility**

This figure shows the time-series data of the VIX and the current volatility (the square root of the current variance defined in the main paper). The current volatility is the historical 22 days actual volatility estimated based on daily return data of the S & P500 index.

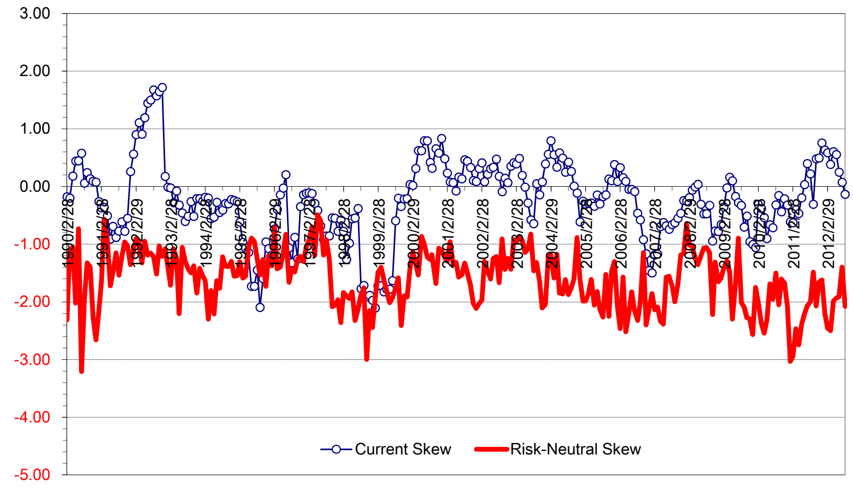

Fig. 10: **The Risk-Neutral Skewness and The Current Skewness**

This figure shows the time-series data of the risk-neutral expected skewness extracted from the SKEW index and the current skewness. The current skewness is the historical 12 months actual skewness estimated based on monthly return data of the S & P500 index.

To illustrate the data, Fig.9 and Fig.10 plot the monthly time series of the risk-neutral expected volatility (VIX), the current volatility (historical 22 days annualized actual volatility), the risk-neutral expected skewness, and the current skewness (historical 12 months actual skewness). Consistent with the theoretical model developed in the previous section and the earlier empirical evidence, the spread between the risk-neutral expected variance (the squared VIX) and the current variance is almost always positive and the spread between the risk-neutral expected skewness and the current skewness is almost always negative. It is interesting that, although the value of the VIX reaches an outstanding peak at the period of the Rehman crisis in 2008, the risk-neutral skewness seems to be more negative at the period of the European financial crisis in 2011 than at the period of the Rehman crisis.

In addition to the variance and skewness risk premiums, we also consider a set of other more traditional predictor variables for the predictive regressions examined in the following subsection. Specifically, we obtain monthly P/E ratios and dividend yields for the S & P 500 directly from Standard & Poor's. Data on the three-month T-bill, the high-yield spread (hys) (between Moody's BAA and AAA corporate bond spreads), and the term spread (ts) (between the ten-year T-bond and the three-month T-bill yields) are taken from the Thomson Reuters Data Stream. The CAY as defined in Lettau and Ludvigson[2001] is downloaded from Lettau and Ludvigson's Web site.

Basic summary statistics for the monthly returns and predictor variables are given in Table 2. The sample period extends from January 1990 to August 2012. All variables are reported in monthly-based percentage form whenever appropriate. The $r_{m,t} - r_{f,t}$ denotes the logarithmic return on the S & P 500 in excess of the three-month T-bill rate. VIX^2 denotes the squared VIX index. ISKew refers to the risk-neutral expected skewness extracted from the CBOE SKEW index by the formula of $ISkew = \frac{1}{10}(100-\text{Skew index})$. CVar and CSkew refer to the current variance, which is the annualized actual variance based on historical 22 days daily return data, and the current skewness, which is the actual skewness based on historical 12 months monthly return data, respectively. vp and skp respectively refer to the variance and skewness risk premiums, that is, $vp \equiv VIX^2 - CVar$ and $skp \equiv ISkew - CSkew$. The predictor variables include the log price-earning ratio ln(pe), the log dividend yield ln(dy), the high yield spread (hys) defined as the difference between Moody's BAA and AAA bond yield indices, and the term spread (ts) defined as the difference between the ten-year and three-month Treasury yields.

The mean excess return on the S & P 500 over the sample equals 0.3 % monthly. The sample means for the VIX^2 and the current (historical 22 days) annualized variance are 6.0 % and 5.0 %, respectively, and the sample means for the risk-neutral expected skewness and the current (historical 12 months) skewness are -1.6 and -0.2, respectively. The numbers for the traditional forecasting variables are all directly in line with those reported in previous studies. In particular, all of the variables are highly persistent with first-order autocorrelations ranging from 0.95 to 0.99.

Table 2: Summary statistics for the monthly returns and predictor variables

	$r_{m,t} - r_{f,t}$	VIX^2	ISkew	CVar	CSkew	vp	skp	ln(pe)	ln(dy)	hys	ts
(A) Summary Statistics											
(1) Mean	0.3 %	6.0 %	-1.6	5.0 %	-0.2	1.0 %	-1.4	3.1	0.7	1.0 %	1.9 %
(2) Std. Dev.	4.4 %	2.3 %	0.5	3.1 %	0.7	1.5 %	0.8	0.4	0.3	0.4 %	1.2 %
(3) Skewness	-0.6	1.6	-0.4	2.7	-0.2	-2.2	0.2	2.3	0.1	3.2	-0.2
(4) Kurtosis	1.1	3.9	-0.2	10.7	0.7	9.9	0.0	7.1	-0.6	12.9	-1.1
(5) AR(1)	0.07	0.85	0.56	0.75	0.91	0.22	0.74	0.95	0.99	0.96	0.97
(B) Correlation Matrix											
$r_{m,t} - r_{f,t}$	1										
VIX^2	0.02	1									
ISkew	0.06	-0.00	1								
CVar	-0.11	0.87	0.03	1							
CSkew	-0.10	-0.06	0.19	-0.03	1						
vp	0.25	-0.25	-0.07	-0.69	-0.03	1					
skp	0.13	0.05	0.48	0.04	-0.77	-0.01	1				
ln(pe)	0.01	0.25	0.00	0.15	0.00	0.08	0.00	1			
ln(dy)	0.08	-0.14	0.11	-0.10	0.09	-0.02	-0.01	-0.35	1		
hys	-0.05	0.64	-0.04	0.67	0.07	-0.38	-0.09	0.16	0.21	1	
ts	-0.02	0.06	0.08	0.05	0.33	-0.02	-0.24	0.28	0.38	0.26	1

The sample period extends from January 1990 to August 2012. All variables are reported in monthly-based percentage form whenever appropriate. The $r_{m,t} - r_{f,t}$ denotes the logarithmic return on the S & P 500 in excess of the three-month T-bill rate. VIX^2 denotes the squared VIX index. ISKew refers to the risk-neutral expected skewness extracted from the CBOE SKEW index by the formula of $ISkew = \frac{1}{10}(100 - \text{Skew index})$. CVar and CSkew refer to the current variance, which is the annualized actual variance based on historical 22 days daily return data, and the current skewness, which is the actual skewness based on historical 12 months monthly return data, respectively. vp and skp respectively refer to the variance and skewness risk premiums, that is, $vp \equiv VIX^2 - CVar$ and $skp \equiv ISkew - CSkew$. The predictor variables include the log price-earning ratio ln(pe), the log dividend yield ln(dy), the high yield spread (hys) defined as the difference between Moody's BAA and AAA bond yield indices, and the term spread (ts) defined as the difference between the ten-year and three-month Treasury yields.

4.3 Main Empirical Findings

Table 3 provides the results of return predictability regressions with the variance and skewness risk premiums. All of our forecasts are based on simple linear regressions of the S & P500 excess returns on different sets of lagged predictor variables. There are two sets of columns with regression estimates. The first set of columns shows OLS estimates by monthly return regressions, that is, one-month-ahead forecasts and the second set shows OLS estimates by non-overlapped quarterly return regressions, that is, one-quarter-ahead forecasts. These regressions are examined in the period from January 1990 to August 2012 and, in particular, each of the monthly return regressions is examined by 270-month samples and each of the quarterly return regressions is examined by 88-quarter samples. Each of the sets of columns consists of five regression results. The first two regressions are one-factor regression models using the variance risk premium (vp-model) or the skewness risk premium (skp-model) as a univariate regressor, while the third regression is two-factor regression model using both the variance and skewness risk premiums (vp+skp-model). The fourth regression model, which is denoted by 3-factor-model, represents the theoretical linear model of (27) derived in the previous section. Finally, we also provide the stepwise-selection model (Stepwise-model) of which the universe of independent variables consists of the risk premiums in higher-order moments, changes of those of risk premiums, and one of the traditional predictor variables, that is, the log price-earning ratio ln(pe). The variables such as $\triangle VIX^2$ and $\triangle ISkew$ exhibited in this table are monthly or quarterly changes of the VIX^2 and $ISkew$, respectively.

Table 3: The Monthly and Quarterly Return Regressions

	(A) Monthly Return Regression Models					(B) Quarterly Return Regression Models				
	vp	skp	vp+skp	3-factor	Stepwise	vp	skp	vp+skp	3-factor	Stepwise
Constant	-0.004	0.013	0.006	-0.004	-0.015	-0.007	0.027	0.016	-0.042	-0.042
(t-stat)	-1.18	2.43**	1.18	-0.42	-1.57	-0.68	1.48	0.91	-1.44	-1.44
VIX^2										
ISkew										
CVar				0.158	0.312				0.909	0.909
				1.37	2.54**				2.43**	2.43**
CSkew										
$\triangle$ VIX^2					-0.883					
					-3.35***					
$\triangle$ ISkew										
$\triangle$ CVar					0.369					
					2.11**					
$\triangle$ CSkew										
vp	0.699		0.704	0.921	1.416	1.529		1.646	2.845	2.845
	4.19***		4.25***	4.02***	4.85***	2.95***		3.17***	4.03***	4.03***
skp		0.007	0.007	0.007	0.008		0.013	0.019	0.018	0.018
		2.11**	2.22**	2.16**	2.42**		1.11	1.60	1.56	1.56
$\triangle$ vp										
$\triangle$ skp										
ln(pe)										
Adj.R^2	5.8 %	1.3 %	7.2 %	7.5 %	10.6 %	8.1 %	0.3 %	9.8 %	14.7 %	14.7 %

The sample period extends from January 1990 to August 2012. VIX^2 denotes the squared VIX index. ISKew refers to the risk-neutral expected skewness extracted from the CBOE SKEW index by the formula of $ISkew = \frac{1}{10}(100-\text{Skew index})$. CVar and CSkew refer to the current variance, which is the annualized actual variance based on historical 22 days daily return data, and the current skewness, which is the actual skewness based on historical 12 months monthly return data, respectively. vp and skp respectively refer to the variance and skewness risk premiums, that is, $vp \equiv VIX^2 - CVar$ and $skp \equiv ISkew - CSkew$. The variables such as $\triangle VIX^2$ and $\triangle ISkew$ exhibited in this table are monthly or quarterly changes of the VIX^2 and the ISkew, respectively. The predictor variables include the log price-earning ratio ln(pe).

From the monthly return regression results in this table, we can find that the slope coefficients of the vp- and skp-model are both significant at 5 % level and, in particular, the slope coefficient of the vp-model is significant at 1 % level. Moreover, the slope coefficients of the vp+skp-model are also significant at the same level with which mentioned above, and this model can account for about 7.2 % of the monthly return variation. The 3-factor-model represents the theoretical implication of (27) and this model has a superior predictive power in the adjusted R^2 than the vp+skp-model due to the additional variable of the current variance (CVar). Although the stepwise-model is not equivalent to the theoretical implication of (27), that is, the 3-factor-model, all the independent variables of CVar, vp, and skp in (27) are significant at 5 % or 1 % level. These results indicate that the theoretical model of (27) and, in particular, the variance and skewness risk premiums have superior predictive power for future aggregate stock market index returns, and this indication is consistent with the theory provided in the previous section in this paper.

The quarterly regressions reported in this table further underscore the significance of the monthly return regressions and, in contrast to the monthly return regressions, all of the t-statistics for the skewness risk premium are insignificant at conventional levels. But, interestingly, we can find that the stepwise-model is perfectly equivalent to the theoretical implication of (27), that is, the 3-factor-model, and this model can account for about 14.7 % of the quarterly return variation. Although the slope coefficient to the skewness risk premium is not significant as mentioned above, the coefficients to the variance risk premium and the current variance are both significant at 5 % level and, in particular, at 1 % level for the variance risk premium.

Table 4: The Univariate Regressions with Traditional Predictor Variables

(A) Monthly Return Regressions								
	ln(pe)	△ ln(pe)	ln(dy)	△ ln(dy)	hys	△ hys	ts	△ ts
Slope Coeff.	0.001	-0.002	0.011	-0.090	-0.516	-1.822	-0.084	-0.513
p-Value (%)	86.7	93.1	22.4	15.1	40.0	38.4	71.2	56.6
Adj.R^2 (%)	-0.4	-0.4	0.2	0.4	-0.1	-0.1	-0.3	-0.3

(B) Quarterly Return Regressions								
	ln(pe)	△ ln(pe)	ln(dy)	△ ln(dy)	hys	△ hys	ts	△ ts
Slope Coeff.	0.008	0.059	0.038	-0.093	-1.118	-7.402	-0.132	-1.661
p-Value (%)	73.8	10.6	20.6	41.3	58.8	2.5**	86.4	31.6
Adj.R^2 (%)	-1.0	1.9	0.7	-0.4	-0.8	4.6	-1.1	0.0

The sample period extends from January 1990 to August 2012. We obtain monthly P/E ratios (pe) and dividend yields (dy) for the S & P 500 directly from Standard & Poor's. Data on the three-month T-bill, the high-yield spread (hys) (between Moody's BAA and AAA corporate bond spreads), and the term spread (ts) (between the ten-year T-bond and the three-month T-bill yields) are taken from the Thomson Reuters Data Stream.

Let us show the other results to emphasize the superiority of the skewness risk premium, as well as the variance risk premium, as a predictor variable for the equity excess return. Table 4 reports monthly- and quarterly-based predictive regression results for the S & P500 index excess returns with each of the traditional predictor variables exhibited in this table, that is, the price-earning ratio (pe), dividend yield (dy), high-yield spread (hys), and term spread (ts) defined in the previous subsection and the changes of those of the variables. As shown in this table, we can find that, in the case of the monthly return regressions, none of the predictor variables are superior in the adjusted R^2 to the variance and skewness risk premiums. In the case of the quarterly return regressions in this table, it seems that there are some variables which have superior adjusted R^2 in comparison with the skewness risk premium, but, none of the variables in this table are superior in the adjusted R^2 to the variance risk premium. (See Table 3)

Table 5: Summary statistics for the CAY

	$r_{m,t} - r_{f,t}$	cay	vp	skp
		(A) Summary Statistics		
(1) Mean	0.29 %	0.21 %	0.94 %	-132.51 %
(2) Std. Dev.	4.38 %	2.38 %	1.55 %	75.83 %
(3) Skewness	-0.59	-0.11	-2.27	0.18
(4) Kurtosis	1.12	-1.40	10.02	0.08
(5) AR(1)	0.08	0.98	0.23	0.75
		(B) Correlation Matrix		
$r_{m,t} - r_{f,t}$	1			
cay	0.09	1		
vp	0.24	0.17	1	
skp	0.14	0.26	0.01	1

The sample period extends from January 1990 to January 2012. The CAY is the aggregate-consumption wealth ratio defined in Lettau and Ludvigson[2001], which is quarterly-based data and downloaded from Lettau and Ludvigson's web site.

Table 6: The Univariate Regressions with the CAY

	cay	vp	skp
(A) Monthly Return Regressions			
	cay	vp	skp
Slope Coeff.	0.157	0.683	0.008
p-Value (%)	17.2	0.0***	2.3**
$Adj.R^2$ (%)	0.3	5.5	1.6
(B) Quarterly Return Regressions			
	cay	vp	skp
Slope Coeff.	0.418	1.618	0.016
p-Value (%)	27.4	0.2***	21.5
$Adj.R^2$ (%)	0.2	9.3	0.7

The sample period extends from January 1990 to January 2012. The CAY is the aggregate-consumption wealth ratio defined in Lettau and Ludvigson[2001], which is quarterly-based data and downloaded from Lettau and Ludvigson's web site. For the monthly return regressions, we define a monthly CAY series from the most recent quarterly observation.

Table 6 reports monthly- and quarterly-based predictive regression results for the S

& P500 index excess returns with the CAY, the aggregate-consumption wealth ratio defined in Lettau and Ludvigson[2001]. The CAY is quarterly-based data and downloaded from Lettau and Ludvigson's web site. The downloaded data covers January 1990 to January 2012. Table 5 shows summary statistics for the CAY as well as the variance and skewness risk premiums under the period from January 1990 to January 2012. For the monthly return regressions, we define a monthly CAY series from the most recent quarterly observation.

As shown in Table 6, we can find that the CAY does not seem to be superior predictor variable in comparison with the variance and skewness risk premiums. This result is similar to the results in Table 4 and also suggests that the skewness risk premium, as well as the variance risk premium, has superior predictive power for future aggregate stock market index returns.

5 Concluding Remarks

In this paper, we study risk premiums in higher order moments of financial asset returns in a general equilibrium setting. Extending the model proposed by Drechsler and Yaron[2011] with a stochastic jump intensity in the processes of both the long-run risk factor and the variance of consumption growth rate, we provide an explicit representation for the variance and skewness risk premiums in a general equilibrium setting. Modeling the stochastic jump intensity endogeneously and deriving a representation of the risk-neutral skewness with that intensity, we propose a possible reason of the empirical fact of time-varying and negative risk-neutral skewness. In particular, we find that the existence of the negative risk-neutral skewness and the skewness risk premium have a close relationship with the existence of the jumps and the jump risk premium, respectively. Moreover, providing an equity risk premium representation of a linear factor pricing model with the variance and skewness risk premiums, we demonstrate the reason why those risk premiums are able to explain a nontrivial fraction of the time series variation in the aggregate stock market returns. Finally, we show an empirical evidence in which the skewness risk premium, as well as the variance risk premium, has superior predictive power for future aggregate stock market index returns.

Some recent studies such as Bali and Hovakimian[2009], Yan[2009], Chang, et.al.[2013], Driessen, et.al.[2012], and Rehman and Vilkov[2012] focus on a significant relationship between skewness or jump risks and expected stock returns, and they provide empirical evidence for a significantly positive link between the expected stock returns and the jump or skewness risks. To the best of our knowledge, this study is the first to provide a theoretical implication in their empirical evidence in terms of the LRR model approach pioneered by Bansal and Yaron[2004]. It remains some challenges for future research on providing an explicit theoretical explanation for the results presented by the recent studies cited above with the theoretical implication shown in this paper. And moreover, it also needs a detailed analysis on the reasons why the skewness and variance risks are priced differently and, in particular, independently of each other. Further insight into this aspect is left to further work.

Appendix A Proof of Proposition 2

From the definition of the variance risk premium (18) and the expressions of the conditional variance of the market return $r_{m,t+2}$ on time $t+1$ under each of the probability measures, we can derive the following expression,

$$\begin{aligned} vp_t = &-B_r^t\Big[\Lambda_{\sigma^2}H_{\sigma^2} + \varphi_\xi(\varphi_\xi\Lambda_q + \rho\varphi_u\Lambda_\lambda)H_q\Big]B_r q_t \\ &+ B_r^t H_{\sigma^2} B_r\Big[\mathbb{E}_t^{\mathbb{Q}}[J^{\mathbb{Q}}_{\sigma^2,t+1}] - \mathbb{E}_t^{\mathbb{P}}[J^{\mathbb{P}}_{\sigma^2,t+1}]\Big] \\ &+ B_r^{2t}\Big[\mathrm{diag}(\psi^{(2)}(-\Lambda))\mathrm{E}_t^{\mathbb{Q}}[\Pi_{t+1}] - \mathrm{diag}(\psi^{(2)}(0))\mathrm{E}_t^{\mathbb{P}}[\Pi_{t+1}]\Big], \end{aligned} \tag{28}$$

where $\Lambda_q \equiv (1-\theta)\kappa_1 A_q$, $\Lambda_\lambda \equiv (1-\theta)\kappa_1 A_\lambda$ (See (13)), and

$$\mathrm{diag}\Big(\psi^{(2)}(-\Lambda)\Big) \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_x^{(2)}(-\Lambda_x) & 0 & 0 & 0 & 0 \\ 0 & 0 & \psi_{\sigma^2}^{(2)}(-\Lambda_{\sigma^2}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathbb{E}_t^{\mathbb{Q}}[\Pi_{t+1}] \equiv \begin{pmatrix} 0 & \mathbb{E}_t^{\mathbb{Q}}[\lambda_{x,t+1}] & \mathbb{E}_t^{\mathbb{Q}}[\lambda_{\sigma^2,t+1}] & 0 & 0 & 0 \end{pmatrix}^t,$$
$$\mathbb{E}_t^{\mathbb{P}}[\Pi_{t+1}] \equiv \begin{pmatrix} 0 & \mathbb{E}_t^{\mathbb{P}}[\lambda_{x,t+1}] & \mathbb{E}_t^{\mathbb{P}}[\lambda_{\sigma^2,t+1}] & 0 & 0 & 0 \end{pmatrix}^t,$$

Substituting the following facts,

$$\mathbb{E}_t^{\mathbb{Q}}[J^{\mathbb{Q}}_{\sigma^2,t+1}] = \lambda_{\sigma^2,t}\psi^{(1)}_{\sigma^2}(-\Lambda_{\sigma^2}),$$
$$\mathbb{E}_t^{\mathbb{P}}[J^{\mathbb{P}}_{\sigma^2,t+1}] = \lambda_{\sigma^2,t}\psi^{(1)}_{\sigma^2}(0),$$

into (28) and considering (6) and (16), we can obtain the representation (21). □

Appendix B The Risk-Free Rate

The explicit expression of the risk-free rate can be obtained by substituting $r_{f,t}$ into $r_{j,t+1}$ in (3). We finally provide the following proposition on the risk-free rate $r_{f,t}$.

Proposition 5 (The Risk-Free Rate) *The risk free rate is expressed as follows in terms of the state variables of σ_t^2, q_t, and λ_t.*

$$r_{f,t} = \beta_{rf,c} + \beta_{rf,x}x_t + \beta_{rf,\sigma}\sigma_t^2 + \beta_{rf,q}q_t + \beta_{rf,\lambda}\lambda_t$$

where

$$\begin{aligned}
\beta_{rf,c} &\equiv -\theta\log\delta + \gamma\mu_g - (\theta-1)(\kappa_0 - A_0) - (\theta-1)\kappa_1(A_0 + A_\sigma\mu_\sigma + A_q\mu_q + A_\lambda\mu_\lambda) \\
\beta_{rf,x} &\equiv \gamma - (\theta-1)A_x(\kappa_1\rho_x - 1) \\
\beta_{rf,\sigma} &\equiv (1-\theta)A_\sigma(\kappa_1\rho_\sigma - 1) - \frac{1}{2}\Big[\gamma^2\varphi_\eta^2 + (\theta-1)^2\kappa_1^2A_x^2\varphi_e^2\Big] \\
\beta_{rf,q} &\equiv (1-\theta)A_q(\kappa_1\rho_q - 1) - \frac{1}{2}(\theta-1)^2\kappa_1^2\Big[A_\sigma^2 + A_q^2\varphi_\xi^2 + 2A_qA_\lambda\varphi_\xi\varphi_u\rho + A_\lambda^2\varphi_u^2\Big] \\
\beta_{rf,\lambda} &\equiv (1-\theta)A_\lambda(\kappa_1\rho_\lambda - 1) \\
&\quad - l_x\Big[\exp(\frac{1}{2}(\theta-1)^2\kappa_1^2A_x^2) - 1\Big] - l_\sigma\Big[\exp(\frac{1}{2}(\theta-1)^2\kappa_1^2A_\sigma^2) - 1\Big]
\end{aligned}$$

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